Inspiring Greatness
INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

College Prep Algebra

April 22, 2020

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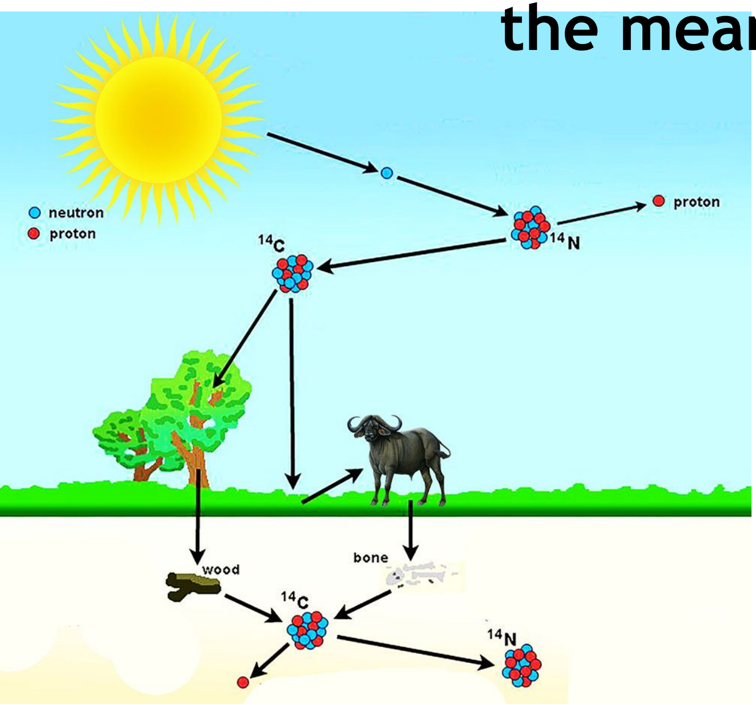
Lesson: April 22, 2020

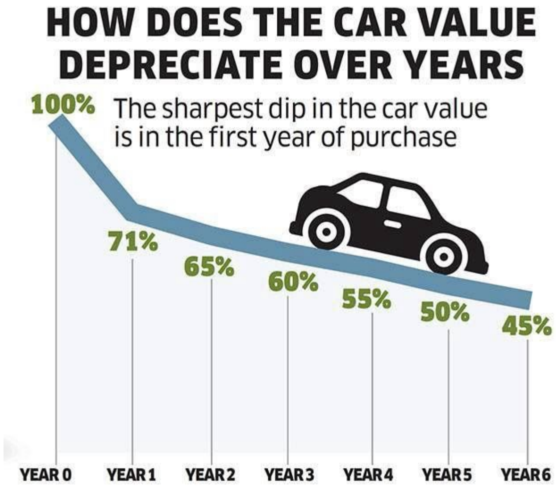
Objective/Learning Target:
How to exponential formulas to solve problems.

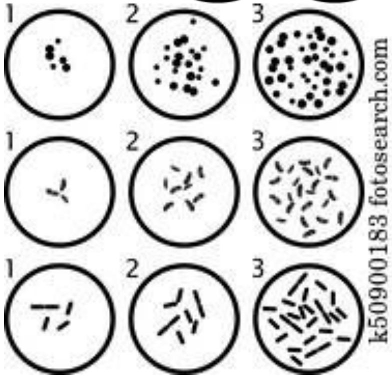
Let’s Get Started:

Exponential decay and growth are all around you, but you may not even be aware of it!

At the end of this lesson, you should be able to figure out the meaning of these pictures.







Lesson:
To apply exponentials, you must have formulas.

The formulas will be given to you on the practice problems.

<u>The one thing about formulas is to remember you must have data for all the variables, except for one!</u>

So if the formula has 7 variables, the problem must give you values for <u>six of the seven</u> variables!

Go and try and practice problems.

Practice: Use the calculator linked here.

Scientific Calculator

Using Exponential Formulas

Answer the questions on a sheet of paper, then review the solutions.

To carbon date fossils, archaeologists measure the amount of carbon 14 is present at the time of discovery.

The number of grams of carbon 14 is given by the formula to the right.

$$A = A_0 e^{-0.000124t}$$

- A is the number of grams of carbon 14 at present times
- A_0 is the number of grams of carbon 14 while alive
- t is the number of years since death

1) How much carbon 14 would there be in an animal's fossil if the animal had 1000 mg of carbon 14 while alive and the fossil is about 13,000 years old?

$A = ?$ $t = 13{,}000$

$A_0 = 1000$

$A = (1000) \cdot e^{(-0.000124 \cdot 13000)}$

$A \approx 199.4$ or 200 mg

2) How much carbon 14 would a live animal have had if the fossil at present has 15 mg of carbon 14 and the fossil is about 10,000 years old?

$A = 15$ $t = 10000$

$A_0 = ?$

$15 = A_0 e^{(-0.000124 \cdot 10000)}$

$\frac{15}{e^{[illegible]}} = \frac{A_0 e^{[illegible]}}{e^{[illegible]}}$

$A_0 = \frac{15}{e^{(-0.000124 \cdot 10000)}} \approx 51.8$

so about 52 mg

Knowing how long it will take for something to double is a popular theme for mathematicians.

The formula for used for questions about doubling is given by the formula to the right.

$$A = A_0(2)^{t/d}$$

- A is the amount at time t
- A_0 is the amount when starting
- t is the amount of time
- d is the time it takes to double

3) In 2002, there were 7.1 million people living in London, England. If the population is expected to double in 2090, what is the expected population for London in 2050

$A = ?$ t = 2002 to 2050 is 48

$A_0 = 7.1$ million d = 2002 to 2090 is 88

$A = 7.1(2)^{48/88}$

$A \approx 10.4$ million

4) At age 22, Collin's starting salary is \$35,000 per year. He doubles his salary every 10 years until he retires at age 68. What is his annual salary when he retires?

$A = ?$ $t = 44$ (age 22 to 68)

$A_0 = 35000$ $d = 10$

$A = 35000(2)^{44/10}$

$A \approx \$73{,}8924$

Radioactive decay and car depreciation are only two examples of half-life that we study.

The formula used for questions about half life is given at the right.

$$A = A_0(0.5)^{(t/h)}$$

- A is the amount at time t
- A_0 is the amount when starting
- t is the amount of time
- h is the half-life, in the time measured as described by t

5) A new car is purchased for $24,000. The half-life for that car is 3 years. How much will that car be worth after 12 years?

$A = ?$ $t = 12$

$A_0 = 24{,}000$ $h = 3$

$A = 24{,}000(0.5)^{12/3}$

$A = \$1500$

6) A radioactive isotope of selenium ^{75}Se used in the creation of medical images of the pancreas, has a half-life of 119.77 days. If 200 milligrams are given to a patient, how many milligrams are left after 30 days?

$A = ?$ $t = 30$

$A_0 = 200$ $h = 119.77$

$A = 200(0.5)^{30/119.77}$

$A \approx 168$ mg

Compound interest happens when interest is paid on both the invested amount and the interest. There are two formulas for finding compound interest shown to the right.

Compound Interest is when the interest is compounded for a typical number of times.

Continuous Compound Interest is used to see what the Maximum total amount could be.

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuous Compound Interest

$$A = Pe^{rt}$$

- A is the amount at the end of t years
- P is the principal, the amount you start with
- r is the percentage rate as a decimal (divide by 100)
- t is the time in years
- n is the number of times compounded in a year

Annual, $n=1$	Monthly, $n=12$
Semiannually, $n=2$	Weekly, $n=52$
Quarterly, $n=4$	Daily, $n=365$

7) If you put $3200 in a savings account that earns 2.5% interest per year <u>compounded quarterly</u>, how much would you expect to have in the account in 3 years?

$A = ?$ $r = \frac{2.5}{100} = .025$ $t = 3$

$P = 3200$ $n = 4$

$A = 3200\left(1 + \frac{0.025}{4}\right)^{4 \cdot 3}$

$A \approx \$3448.42$

8) How much money would you need to put in a savings account now so that you would have $32,000 in 18 years? The account earns 1.7% compounded weekly.

$A = 32000$ $r = \frac{1.7}{100} = .017$ $t = 18$

$P = ?$ $n = 52$

$32000 = P\left(1 + \frac{0.017}{52}\right)^{52 \cdot 18}$

$\frac{32000}{stuff} = \frac{P \cdot stuff}{stuff}$

$P = \frac{32000}{\left(1+\frac{0.017}{52}\right)^{52 \cdot 18}} \approx \$23565.55$

9) A banker wants to know what the **maximum** amount of money he would have to pay on an account that starts with $500 after 25 years at a rate of 1.2%

$A = ?$ $r = \frac{1.2}{100} = 0.012$

$P = 500$ $t = 25$

$A = 500e^{0.012 \cdot 25}$

$A \approx \$674.93$

10) A banker does not want to pay more than $80,000 on an account after 18 years at 2.3%. What would be the maximum starting principal the banker would allow?

$A = 80000$ $r = \frac{2.3}{100} = 0.023$

$P = ?$ $t = 18$

$80000 = P \cdot e^{0.023 \cdot 18}$

$\frac{80000}{STUFF} = \frac{P \cdot STUFF}{STUFF}$

$P = \frac{80000}{e^{0.023 \cdot 18}} \approx \$52880.08$

Additional Practice

Compound Interest Formula